

A Data-Driven Methodology for Dynamic Pricing and Demand Response in Electric Power Networks

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Abstract

The practice of disclosing price of electricity before consumption (dynamic pricing) is essential to promote aggregator-based demand response in smart and connected communities. However, both practitioners and researchers have expressed the fear that wild fluctuations in demand response resulting from dynamic pricing may adversely affect the stability of both the network and the market. This paper presents a comprehensive methodology guided by a data-driven learning model to develop stable and coordinated strategies for both dynamic pricing as well as demand response. The methodology is designed to learn offline without interfering with network operations. Application of the methodology is demonstrated using a sample 5-bus PJM network. Results show that it is possible to arrive at stable dynamic pricing and demand response strategies that can reduce price of electricity as well as improve network load balance.

Keywords: Dynamic pricing, aggregated demand response, electric power network, Bayesian demand prediction.

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1. Introduction

Grid modernization will continue to enhance timely communication among the system operator (SO), producers, and load aggregators. This will provide the infrastructure to implement dynamic pricing of electricity and increase demand response (DR) participation. The phrase *dynamic pricing* has various interpretations in practice and is used to refer to policies like critical peak pricing, real-time pricing and many variants thereof. In this paper, the term *dynamic pricing* is used to refer to the practice of offering binding price of electricity just ahead of consumption. Whereas, the term DR will refer to the load scheduling action by aggregators based on price of electricity and preferences of the consumers in the households, businesses, and industries. DR has long been recognized as an effective mechanism to improve functioning of power network operation. Implementation of full fledged DR programs will require appropriate pricing practices [1]. Dynamic pricing has been identified as a key to promote DR, early in the millennium [2] and more recently [3, 4]. However, till date, dynamic pricing policies have remained limited in the U.S. to time of use (TOU) pricing, critical peak pricing (CPP), ex-post real time pricing (RTP), among others. Many pilot projects across three continents in recent years have examined the benefits of dynamic pricing on load balancing and consumer cost reduction [5, 6, 7].

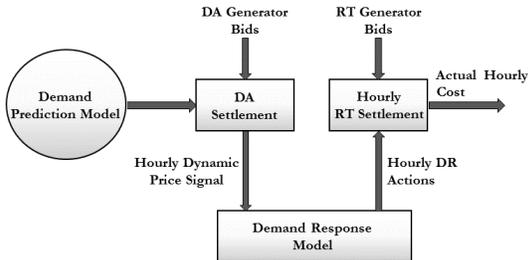


Fig. 1. Dynamic pricing and DR framework

In this paper, we develop a comprehensive methodology to design complimentary policies for dynamic pricing decision by the system operator and the DR actions by the load aggregators, while considering both market and network characteristics. The framework of our methodology is depicted in Fig. 1. The load

aggregators predict the day-ahead (DA) hourly demand by applying a Bayesian model

on historical demand data (prior) and the most recent hourly demand (based on DR actions). It is assumed that the load aggregators are price takers and submit the predicted DA quantities as demand bids to the system operator (SO). Using these demand bids and the DA price bids submitted by the generators, the SO obtains a DA schedule through a network constrained least cost dispatch approach. This yields DA hourly quantities and corresponding LMPs. Now, before each hour of actual dispatch, the SO determines the binding nodal dynamic prices and offers those to the aggregators. The SO obtains these dynamic prices using an exponential learning model with two inputs: the DA price and the previous day's dynamic price for the current hour. The aggregators at the load nodes decide their DR actions (consumption plan) for the current and the remaining hours of the day using the binding dynamic prices for the current hour and the predicted dynamic prices for the future hours, which they obtain using a regression model. Any deviation in load consumption for the current hour from the DA demand bid is settled using the real-time (RT) prices. It is considered that the generators submit separate RT bids to the SO, and the SO settles the market to pay the generators for the hour using a two-settlement (DA and RT) approach. This determines the cost to the SO for satisfying the demands. Note that, the SO's revenue depends on the binding dynamic prices and the corresponding DR decisions by the aggregators, which in turn determines the cost to the SO. Hence, if the dynamic prices are not designed properly, the resulting DR actions may lead to imbalance in the revenue and cost for the SO. Our model is designed to obtain stable dynamic pricing and demand response policies to improve load balance and reduce cost to the consumers, while maintaining revenue neutral status of the SO. A recent paper [8] addresses a similar problem using a closed loop dynamical system model and derives stable ex-ante pricing strategy for each interval of a network operation and the corresponding DR strategies. Our methodology offers a data-driven learning approach that is granular and can consider different

types of load, their operating preferences, network constraints, and a two-settlement approach for the market.

Literature shows that availability of smart technology (eg., advanced metering infrastructure, AMI) increases price elasticity and grid efficiency [9, 10]. Beneficial impacts of dynamic pricing have been explored via seventy-four pricing experiments across three continents during the last decade [5]. The results in these studies show that in the presence of higher price peaks better benefits of load balancing from dynamic pricing. It is shown in [7] that dynamic pricing can achieve a peak demand reduction of 10-14%, customer cost reduction of 2-5%, and a social welfare increase by \$141-\$403 million in a year. A study conducted in California [6] found that most consumers will benefit from dynamic pricing, and also that low income consumers will not be impacted negatively, a concern that was expressed earlier. The study presented in [11] claimed that a careful design of dynamic pricing schemes is needed to increase the consumer flexibility. A balanced RTP strategy was proposed in [12] where the consumers are offered binding prices ahead of consumption (ex-ante prices). Need to understand consumer behavior in a market to address volatility between the ex-ante and ex-post prices, inherent in networks with RTP and DR, is emphasized in [13]. It was argued in [14, 15] that if the RTP and the corresponding aggregator-guided DR strategies are not aligned properly through design, there may be greater peaks in demand than normal condition, which might instigate worst outcomes like blackouts.

As reported in [2], even a slight DR participation may significantly reduce the wholesale electricity prices. It is claimed that DR assisted balancing of consumer demand and resulting increase in load factor will ease the burden of current practice of maintaining peaking generation capacity at the level of 10 - 15% of the expected demand [16]. In recent years, aggregator-guided DR has been proposed [17]. Aggregators represent interests of large groups of consumers (households, factories, and businesses) by

monitoring and scheduling loads to minimize cost. With emerging home and building energy management systems [18, 19], even individual consumers are being empowered to engage in DR. Proliferation of smart grid with smart appliances will continue to extend the potential of DR [20]. Aggregator-guided DR is claimed to improve users' rationale in the event of price changes, however, true aggregator behavior is yet to be investigated [21]; the authors present an iterative approach to design real time pricing algorithms to minimize the peak-to-average demand ratio. Other approaches adopted for aggregator coordinated DR can be found in [22, 23]. A dynamic energy management framework to simulate DR and to estimate consumer behavior is proposed in [24]. Similar work with minimization of electricity price as a DR objective, can be found in [25, 26]. To our knowledge, most of openly available methods for real time pricing and DR are developed using limited history of price and consumption data. Some of the recent pilot studies that model data to demonstrate potential DR benefits are [27, 28].

The remainder of this paper is organized as follows. The complete methodology is described in Section 2. Section 3 contains a numerical demonstration of the methodology on a modified 5-bus PJM network. Concluding remarks are offered in section 4.

2. Dynamic pricing & DR Methodology

Fig. 2 shows the model elements of the dynamic pricing & DR methodology. In what follows, we provide details of each model.

2.1. Bayesian DA demand prediction model

The aggregator at each load node submit their fixed hourly DA demand bids to SO, which are predicted using the model given here. Let K denote the number of nodes (or buses) in the network. At any node $k \in \{1, \dots, K\}$ and hour $t \in \{1, \dots, 24\}$, let $x_{1,(k,t)}, x_{2,(k,t)}, \dots, x_{n,(k,t)}$ denote the historical hourly load data for the demand random

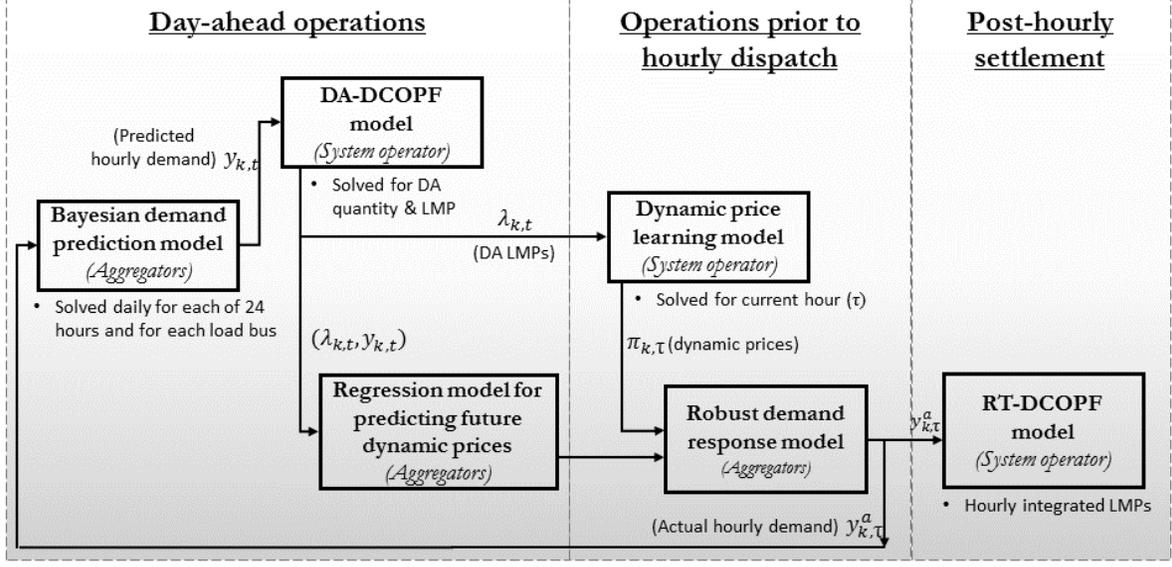


Fig. 2. Elements of dynamic pricing and DR methodology

variable $X_{k,t}$. We assume that this data is characterized by a normal distribution $N(\mu_{k,t}, \sigma_{k,t}^2)$, which is thus the prior $f(x_{k,t})$ for the demand $X_{k,t}$.

Let, for each hour of a day, the aggregator's DR action yields an actual demand denoted by $y_{k,t}^a$. Then we can write the random variable for the actual demand generated by DR action $Y_{k,t}^a$ as $Y_{k,t}^a | (X_{k,t} = x_{k,t}) \sim N(x_{k,t}, \sigma_{k,t}^2)$. We assume that the variance of the demand resulting from DR action is same as that of the prior. Hence, the likelihood for the observed data $y_{k,t}^a$ can be written as

$$L(x_{k,t} | y_{k,t}^a) = f(y_{k,t}^a | x_{k,t}) = (2\pi\sigma_{k,t}^2)^{-1/2} \exp \frac{-1}{2\sigma_{k,t}^2} (y_{k,t}^a - x_{k,t})^2 \quad \forall t \in \{1, \dots, 24\}, \forall k \in \{1, \dots, K\}. \quad (1)$$

Using Bayes' theorem, the posterior distribution (which is proportional to the likelihood times the prior) can be written as,

$$f(x_{k,t} | y_{k,t}^a) \propto f(y_{k,t}^a | x_{k,t}) \cdot f(x_{k,t}) \propto \exp \frac{-1}{2\sigma_{k,t}^2} (y_{k,t}^a - x_{k,t})^2 \cdot \exp \frac{-1}{2\sigma_{k,t}^2} (x_{k,t} - \mu_{k,t})^2. \quad (2)$$

The posterior distribution of the normal conjugate prior is also a normal distribution, the mean of which can be written as $E[X_{k,t} | y_{k,t}^a] = \arg \max_{x_{k,t}} \ln f(x_{k,t} | y_{k,t}^a)$. The posterior mean can then be obtained, by equating to zero the first order partial derivative

of the log-likelihood of $f(x_{k,t}|y_{k,t}^a)$ with respect to $x_{k,t}$, as follows:

$$\frac{\partial \ln f(x_{k,t}|y_{k,t}^a)}{\partial x_{k,t}} = \frac{1}{\sigma_{k,t}^2}(y_{k,t}^a - x_{k,t}) + \frac{-1}{\sigma_{k,t}^2}(x_{k,t} - \mu_{k,t}) = 0. \quad (3)$$

The above yields $E[X_{k,t}|y_{k,t}^a] = \frac{y_{k,t}^a + \mu_{k,t}}{2}$. Variance of the posterior distribution $\text{Var}(X_{k,t}|y_{k,t}^a)$

is inverse of the Fisher information $I(X_{k,t}|y_{k,t}^a)$, which is obtained as,

$$I(X_{k,t}|y_{k,t}^a) = -E \left[\frac{\partial^2 \log f(x_{k,t}|y_{k,t}^a)}{\partial x_{k,t}^2} \right] = - \left(-\frac{1}{\sigma_{k,t}^2} - \frac{1}{\sigma_{k,t}^2} \right) = \frac{2}{\sigma_{k,t}^2}. \quad (4)$$

Hence, the posterior distribution is $N\left(\frac{\mu_{k,t} + y_{k,t}^a}{2}, \frac{\sigma_{k,t}^2}{2}\right)$. The mean hourly DA predicted

demand vector for any node k is given by $\mathbf{y}_k = \left(\frac{\mu_{k,1} + y_{k,1}^a}{2}, \frac{\mu_{k,2} + y_{k,2}^a}{2}, \dots, \frac{\mu_{k,24} + y_{k,24}^a}{2}\right)$. At

the end of each day, the hourly actual demands resulting from the DR actions are added to the historical load data and are used to obtain new prior distribution parameters $(\mu_{k,t}, \sigma_{k,t}^2)$ for the next day. Essentially, the aggregators update their prediction model for the next day based on their DR actions today.

2.2. Day-ahead settlement model

Based on the generators' DA supply offers ($C(P_{ki})$) and the aggregators' DA demand bids ($y_{k,t}$), the SO solves a network constrained least cost dispatch model to obtain the locational marginal prices (LMPs) for each node at every hour of the following day. Let G_k represent the set of generators at node k . We assume that the generators' supply offer constitute quadratic functions given as $C(P_{ki}) = a_{ki}P_{ki}^2 + b_{ki}P_{ki}$, $\forall i \in G_k, \forall k \in \{1, \dots, K\}$, where P_{ki} is real power output supplied at node k by generator i . The DA settlement problem is formulated as a DCOPF [29] that minimizes the cost of supply offers, while meeting the supply-demand balance and other network constraints.

$$\min \sum_{k=1}^K \sum_{i \in G_k} a_{ki}P_{ki}^2 + b_{ki}P_{ki}. \quad (5)$$

The dual variables from the power balance and the line flow constraints are decomposed into three components namely, marginal cost of energy at reference buses, marginal cost of losses, and the cost of congestion [30]. Using these, the LMPs ($\boldsymbol{\lambda}_k$) for all nodes are calculated. Solution of DCOPF for each hour yields $\boldsymbol{\lambda}_k = \{\lambda_{k,t}\}$, the set

of hourly DA LMPs for each node.

2.3. Regression model for predicting dynamic prices

The DR actions for each hour taken by the aggregators use the binding dynamic prices for the current hour and the predicted dynamic prices for the remaining hours of the day, which they obtain using the regression model. A piecewise linear regression equation is developed for each load node k with hourly demand as the independent variable and the corresponding cost (demand \times DA price) as the dependent variable. We use a data set containing the current DA hourly demand-price pairs (D_k) and historical DA hourly demand-price pairs (D_k^h) for a number of days (say, H) to build the regression model. Note that $D_k^h = \{(y_{k,t}^h, \lambda_{k,t}^h) : \forall h \in \{1, 2, \dots, H\}, \forall t \in \{1, 2, \dots, 24\}\}$ and $D_k = (y_{k,t}, \lambda_{k,t})$. Considering a m piecewise components for the regression model, we maximize a set of (up to m) affine functions by minimizing the least square. Let U denote the $(H + 1) \times 24$ demand-price pairs in the data set used in the regression model. Denoting each demand-price pair as (y_u, λ_u) for $u \in U$, we write the following model to optimize the affine functions.

$$\min_{\beta_0^i, \beta_1^i} \sum_{u \in U} \left[\max_{i=1, \dots, m} (\beta_0^i + \beta_1^i y_u) - y_u \lambda_u \right]^2. \quad (6)$$

The solution of the above yields an optimal set of l affine functions ($l \leq m$) and the corresponding l non-overlapping partitions of U . For more details on this approach, see [31].

2.4. A learning model for hourly dynamic prices

The SO obtains the nodal dynamic prices for each hour of the day by applying a learning model (as in [32]) on the previous day's dynamic price and the DA price. Let $\pi_{k,t}$ denote the dynamic price offered for node k at hour t , and $\gamma > 0$ denote the learning rate. Then, using \hat{t} to denote the same hour t on the previous day, we write that

$$\pi_{k,t} = \gamma \lambda_{k,t} + (1 - \gamma) \pi_{k,\hat{t}}. \quad (7)$$

2.5. A robust DR model

We first describe the types of loads that are managed by the aggregators. Loads are either non-deferrable (fixed loads) or deferrable (shiftable and shift-adjustable loads). Deferrable loads are managed by the aggregators based on user preferences. Load curtailment is not considered as an option for aggregators in this model.

Non-deferrable load: These loads are fixed and have specified operation times and energy consumption levels. Let C denote the set of load consuming entities (e.g., homes, businesses, and factories) managed by an aggregator. For $i \in C$, let F_i denote the set of fixed loads and for each $j \in F_i$, f_{ijt} denotes the energy consumed during hour t .

Deferrable load: A deferrable load is characterized by its power consumption level, total duration of operation, and the interval(s) of time when it can be scheduled. These loads are further classified as shiftable (that are operated with rated power and have flexible hours of operation) and shift-adjustable (for which the power consumption levels can be adjustable along with the hours of operation). We denote the set of shiftable loads for load consuming entity i by S_i . For load $j \in S_i$, the consumption level is s_{ij} and the length of operation is τ_{ij} . The time window within which the operation can be scheduled is $[\underline{T}_{ij}^s, \overline{T}_{ij}^s]$. Hence, if x_{ijt} denote a binary variable indicating on/off status of a shiftable load at time t , we can write the operating constraint as

$$\sum_{t=\underline{T}_{ij}^s}^{\overline{T}_{ij}^s} x_{ijt} = \tau_{ij}, \quad \forall i, j. \quad (8)$$

The set of shift-adjustable loads for an entity $i \in C$ is A_i . The rated (maximum) power consumption per unit time of the individual loads $j \in A_i$ is a_{ij} , which can be lowered up to a prescribed threshold \underline{a}_{ij} . The feasible operating time window of load $j \in A_i$ is $[\underline{T}_{ij}, \overline{T}_{ij}]$. Let u_{ijt} be a binary variable indicating the on/off status of $j \in A_i$ at time t , and α_{ijt} be a continuous variable indicating the level of power consumption. Then we

can write that

$$\underline{a}_{ij} \leq \alpha_{ijt} a_{ij} \leq a_{ij}, \quad \forall i, j, t. \quad (9)$$

Denoting the total energy required to complete operation as E_{ij} , we can write

$$\sum_{t=\underline{T}_{ij}}^{\bar{T}_{ij}} u_{ijt} \alpha_{ijt} a_{ij} = E_{ij}, \quad \forall i, j. \quad (10)$$

Note the above constraint is bi-linear in $u_{ijt} \alpha_{ijt}$. We linearize (10) by incorporating u_{ijt} in (9). Hence, we have

$$u_{ijt} \underline{a}_{ij} \leq \alpha_{ijt} a_{ij} \leq u_{ijt} a_{ij}, \quad \forall i, j, t, \quad (11)$$

$$\sum_{t=\underline{T}_{ij}}^{\bar{T}_{ij}} \alpha_{ijt} a_{ij} = E_{ij}, \quad \forall i, j. \quad (12)$$

Hence the total load scheduled by the aggregator at time t can be written as

$$y_t^a = \sum_{i \in C} \left[\sum_{j \in F_i} f_{ijt} + \sum_{j \in S_i} s_{ij} x_{ijt} + \sum_{j \in A_i} \alpha_{ijt} a_{ij} \right].$$

The aggregator finds y_t^a by solving an optimization model, described below, at each node at the top of each hour. The model uses 24-hour day as the planning horizon. For the current hour τ , based on the binding dynamic price π_τ , an aggregator determines the actual energy consumption for the current hour (y_τ^a) and the planned consumption for the remaining hours $t \in \{\tau + 1, \dots, 24\}$. The model uses the predicted values of the dynamic prices for the remaining hours from the piecewise regression model presented in Section 2.3. We note that these predicted values could differ from the dynamic prices offered by SO in the remaining hours. Such price variations present a risk of ineffective load scheduling. To reduce the impact of this risk, we adopt a robust optimization approach.

Let π_t^{\max} denote the chosen upper $100 \times (1 - \alpha)\%$ confidence bound of the historical values of the dynamic price for each hour t . These bounds are revised each day by updating the data history. Let Γ denote the parameter (in percent) for the degree of robustness, where Γ is 0% when price variations are not considered for any of the remaining hours, and Γ is 100% when price variations are considered for all of the

remaining hours, which yields the most conservative solution. For example, $\Gamma = 80\%$ means that the model considers price variations for $0.8 \times [24 - \tau]$ of the remaining hours. The robust DR model is presented next. For notational simplicity, we write the regression parameters as β_0 and β_1 (without their piecewise index i).

$$\min \pi_\tau y_\tau^a + \sum_{t=\tau+1}^{24} (\beta_0 + \beta_1 y_t^a) + \max_{\substack{z \leq \Gamma, \\ 0 \leq z \leq 1}} \sum_{t=\tau+1}^{24} [\pi_t^{\max} y_t^a - (\beta_0 + \beta_1 y_t^a)] z. \quad (13)$$

By duality principle as described in [33], we can rewrite the objective function as

$$\min \pi_\tau y_\tau^a + \sum_{t=\tau+1}^{24} (\beta_0 + \beta_1 y_t^a) + z\Gamma + \sum_{t=\tau+1}^{24} \zeta_t, \quad (14)$$

s.t.,

$$\text{Constraints (8), (11), \& (12),} \quad (15)$$

$$z + \zeta_t \geq \pi_t^{\max} y_t^a - (\beta_0 + \beta_1 y_t^a), \quad \forall t \in \{\tau + 1, \dots, 24\}, \quad (16)$$

$$\zeta_t \geq 0, \quad y_t^a \geq 0, \quad \forall t \in \{\tau + 1, \dots, 24\}, \quad (17)$$

$$z \geq 0, \quad y_\tau^a \geq 0. \quad (18)$$

In the above model, the numerical values of β_0^i and β_1^i for the decision variable y_t^a are obtained using minimizing piecewise linear cost functions formulation.

2.6. Real-time settlement model

Once the aggregators take DR actions, variation between the DA demand and the actual consumption is settled by the SO using the real-time price bids in the DCOPTF model. This two-settlement process yields the integrated LMPs, which we refer to as settled price in the numerical implementation study. Note that, the total payment made by the SO to the generators could be different from the total revenue collected from the aggregators. However, as demonstrated via numerical experiments, under stable dynamic pricing and DR strategies, the average difference between the cost and the revenue reduces to at or near zero. The SO (a non-profit agent) should neither accumulate excess revenue nor incur deficit.

3. Numerical implementation

In this section, we demonstrate our methodology by implementing it on a sample network with congestion as described below. We conducted our numerical study in two parts. First, for given load compositions and generator bids in DA and RT markets, we determine stable dynamic pricing and DR strategies. Thereafter, we examine the convergence of price and the corresponding load balancing profile over 24 hours, and also analyze the impact of the percentage of deferrable loads in the network on the average dynamic price.

3.1. Sample network: A modified PJM 5-bus system

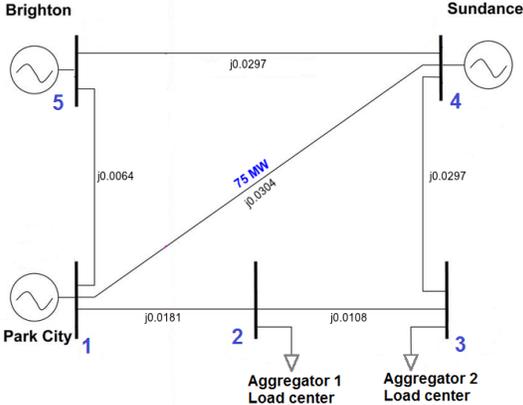


Fig. 3. Modified 5-bus PJM network

We use a minor variant of the PJM 5 bus system with 3 generators, 2 load nodes, and 6 transmission lines as shown in Fig. 3. The parameters a and b of the generators' DA supply offers (see Section 2.2) are $(0.009, 47)$, $(0.007, 35)$, and $(0.005, 10.25)$, representing high, medium, and low cost generators at nodes 1, 5, and 4, respectively; corresponding parameter values for the real-time bids are $(0.04, 80)$, $(0.035, 70)$ and $(0.03, 60)$. The maximum generating capacities are 2000, 2000 and 600 MW for generators at nodes 1, 4, and 5. The reactance of the transmission lines are as marked (see Fig. 3). We consider that the capacity of line connecting buses 1 and 4 is limited to 75 MW, and the remaining lines are unconstrained. The two load nodes are located at buses 2 and 3. In each of these load nodes, DR decisions are made by the respective aggregators.

We make the following assumptions about the load composition of the network.

The loads consist of entities and each entity comprises a number of households and businesses. There are 60 and 40 entities in node 2 (managed by aggregator 1) and node 3 (managed by aggregator 2), respectively. Each entity has non-deferrable (fixed) and deferrable (shiftable and shift-adjustable) loads. It is assumed that distribution of the hourly consumption levels of the fixed loads for all entities at a node can be approximated by a normal distribution. Fig. 4 depicts the mean and the one standard deviation confidence bounds of the hourly fixed loads at node 2. Similar load pattern is considered for fixed loads at node 3. As seen from the figure, the time of operation of the fixed loads are chosen to mimic the two-peak load pattern commonly observed in power networks.

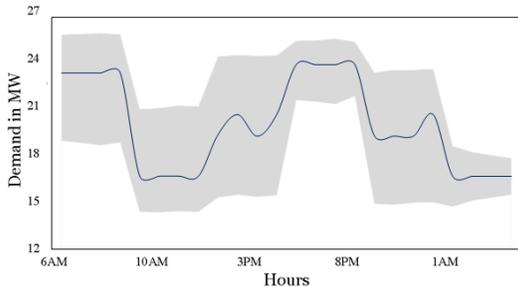


Fig. 4. Fixed load pattern for aggregator 1 at load node 2

There are three shiftable loads for each entity under aggregator 1, whose consumption levels are 2 MW/h, 3.7 MW/h and 5 MW/h. Each of these shiftable loads runs for a total 3 hours a day, where the hours could be non-contiguous. Shiftable load levels managed by aggregator 2 are 1.5 MW/h, 2 MW/h, 3.7 MW/h and 5 MW/h, each with a 3-hour non-contiguous operating time. All entities under both aggregators are considered to have two shift-adjustable loads. When operating, the maximum (minimum) energy consumption levels per hour for these loads are 1.5 (1.05) MW and 5 (3.5) MW. These loads can be run during any non-contiguous hours of the day as long as the total consumption per day reaches 6 MWh and 20 MWh.

3.2. *Discussion of numerical results*

Since DA demand prediction is critical to the success of the dynamic pricing and DR methodology, we first examine the performance of the Bayesian prediction model. The predicted demand is used by the SO to determine the DA prices, which in turn

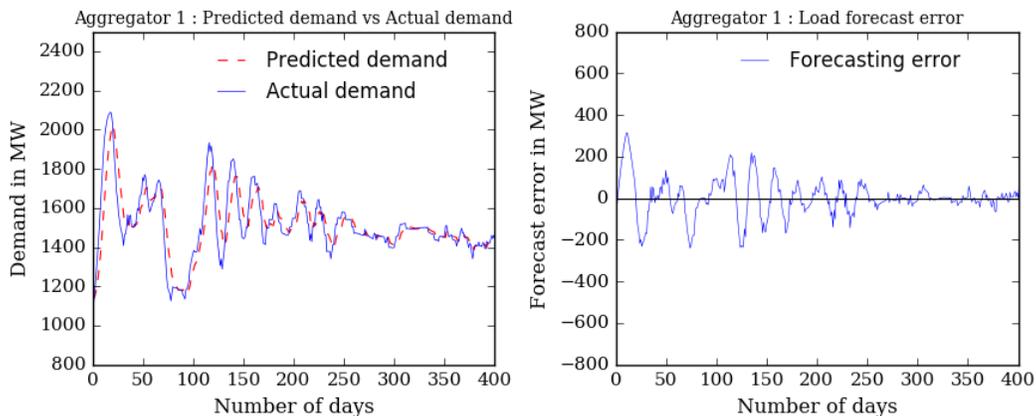


Fig. 5. Performance of Bayesian model for aggregator 1 at hour 10 AM

help to yield the hourly dynamic prices. We then discuss the efficacy of the estimation process for π_t^{max} , which is a key parameter for the robust DR model. In the remaining part of this section, we discuss the evolution of the dynamic pricing and DR strategies over the offline learning period of the methodology, followed by a brief sensitivity study.

Fig. 5 demonstrates the performance of the Bayesian model in predicting the DA demand for aggregator 1. The model predicts hourly demands for all 24 hours, of which the data for hour 10 AM is shown in the figure. It can be observed that during the initial days of simulated learning, due to aggregator’s DR actions, the predicted DA demand vary significantly from the actual demand. As the variations in actual demand due to DR actions subside, the prediction error reduces to a very small value. A very similar behavior is also observed for the demand prediction by aggregator 2.

Before discussing the evolution of prices and consumption pattern in the network, we describe a key parameter of the robust DR model, π_t^{max} . It is the $100 \times (1 - \alpha)\%$ upper bound of the distribution of the historical values of dynamic prices π_t , and is the maximum possible price at a future hour t . This parameter plays a critical role in setting the level of risk considered by the robust DR model. Hence, a proper choice of π_t^{max} is essential to control how conservative we desire our DR actions to be. Unlike robust models commonly found in the literature, we continuously update π_t^{max}

at every iteration of the offline learning using newly available dynamic price data. At the beginning of the learning period, there is no available history of dynamic prices for the network and hence no initial estimate of π_t^{max} .

Therefore, for learning on day 1, we generate a history by drawing samples from a normal distribution with the DA price of day 1 as the mean and an assumed standard deviation (\$4.0/MWh). The $100 \times (1 - \alpha)\%$ bound of this generated history is considered to be the π_t^{max} . On day 2 onwards, the dynamic price of the

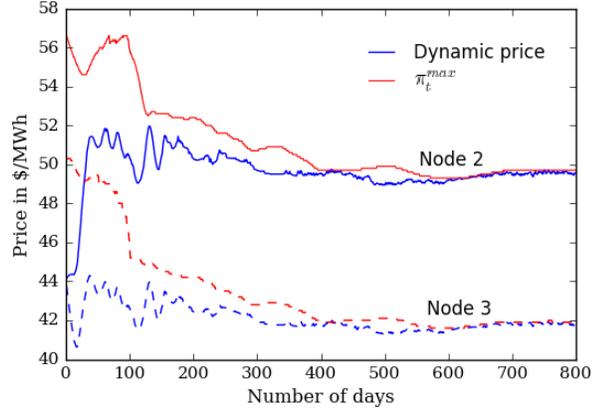


Fig. 6. Trend of π_t^{max} at hour 11 AM

preceding day is added to the history and π_t^{max} is recalculated. Fig. 6 depicts the evolution of π_t^{max} for an arbitrarily chosen hour (11 AM) over the days of simulated learning. As expected, with learning, π_t^{max} approaches the dynamic price π_t . The choice of this initial value of standard deviation is not critical since it is updated as more data is available with the progress of the learning process.

3.2.1. Day-ahead, dynamic, and settled prices for the sample network

The simulated learning using our methodology is continued for a sufficiently large number of days until stable strategies arise with daily average dynamic prices sufficiently close to the daily average settled prices (DA and RT integrated LMPs). The parameter values used in our implementation are: $\alpha = .01$, $\gamma = 0.05$, and $\Gamma = 100\%$.

Fig. 7 depicts the evolution of the daily averages of day-ahead, dynamic, and settled prices at nodes 2 and 3 for over 600 days of simulation. The simulation was run for up to 1200 days, however, changes in prices are negligible after 600 days. As the two load nodes have different load characteristics and the network is subjected to congestion,

the price trajectories, as expected, are distinctly different. It can be seen from the figure that the performance of our methodology is not dependent on the choice of the dynamic prices for day 1. We chose, somewhat arbitrarily, identical starting dynamic prices (\$44/MWh) for both nodes.

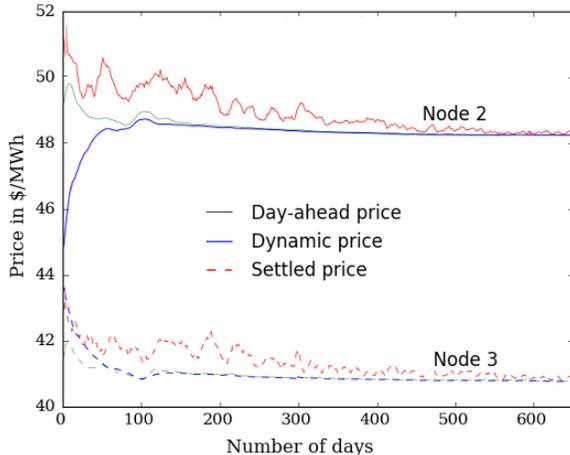


Fig. 7. Evolution of daily average values of day-ahead, dynamic, and settled prices

As observed, during the initial days of learning with limited available data, the prices vary significantly. This variation is caused by wide alteration in DR actions. We observed from our data, gathered during learning, that initial variations in DR produced higher demand peaks than usually observed in networks without dynamic pricing and DR. This confirms the fear that has been expressed in the literature [14]. However, as the offline learning of the dynamic prices continues over a sufficiently large number of days, the SO and the aggregators are able to learn stable and consistent dynamic pricing and corresponding DR strategies, respectively. This is manifested in the convergence of the day-ahead, dynamic, and settled prices. For example, after learning, the standard error (SE) between the settled and the dynamic prices reaches as low as 0.09%. Our methodology demonstrates that a real-world power network that implements dynamic pricing and aggregator guided demand response will not have to suffer through the long periods of price and demand uncertainties, and it can begin its operation with model-guided stable strategies. Table 1 shows the actual values of the stabilized hourly prices for both nodes.

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Table 1 Hourly stabilized price for nodes 2 and 3

Hours	Node 2 (Aggregator 1)					Node 3 (Aggregator 2)				
	DAP \$/MWh	DP \$/MWh	SP \$/MWh	SP-DAP \$/MWh	SE(SP-DAP) \$/MWh	DAP \$/MWh	DP \$/MWh	SP \$/MWh	SP-DAP \$/MWh	SE(SP-DAP) \$/MWh
1	50.88	50.85	50.83	0.05560	0.12058	42.97	43.07	42.90	0.07821	0.30115
2	50.77	50.80	50.90	-0.13115	0.25232	42.90	42.91	42.93	-0.03059	0.56884
3	50.33	50.37	50.47	-0.13407	0.37648	42.50	42.57	42.72	-0.22751	0.75941
4	49.76	49.82	50.11	-0.35533	0.37592	41.97	42.01	42.05	-0.07857	0.69201
5	49.52	49.45	49.67	-0.15366	0.34717	41.78	41.96	42.10	-0.31128	0.49502
6	49.37	49.43	49.53	-0.15506	0.28856	41.66	41.62	41.63	0.03509	0.36658
7	49.29	49.21	49.30	-0.00453	0.15167	41.60	41.57	41.47	0.13486	0.28704
8	49.04	49.02	49.05	-0.01082	0.03019	41.38	41.40	41.52	-0.13367	0.06298
9	48.66	48.66	48.63	0.02757	0.07403	41.05	41.05	40.99	0.05815	0.05348
10	48.41	48.39	48.47	-0.06391	0.07190	40.84	40.85	40.88	-0.04590	0.06844
11	47.75	47.78	47.79	-0.03892	0.02081	40.29	40.30	40.35	-0.06450	0.04862
12	47.09	47.10	47.09	-0.00015	0.01648	39.71	39.72	39.71	-0.00124	0.00496
13	48.36	48.36	48.35	0.00215	0.00998	40.74	40.74	40.74	0.00064	0.00237
14	49.25	49.25	49.25	0.00018	0.00884	41.55	41.54	41.55	0.00023	0.00326
15	49.07	49.08	49.07	0.00411	0.00966	41.39	41.38	41.39	0.00003	0.00260
16	49.17	49.18	49.18	-0.00250	0.00929	41.47	41.47	41.47	0.00014	0.00111
17	46.21	46.21	46.22	-0.01254	0.02249	38.96	38.96	38.94	0.01301	0.01783
18	46.18	46.17	46.17	0.00922	0.02895	38.93	38.92	38.93	0.00056	0.00252
19	46.18	46.18	46.19	-0.00821	0.02777	38.93	38.94	38.93	0.00046	0.00374
20	46.12	46.12	46.12	0.00187	0.01223	38.88	38.89	38.88	0.00022	0.00351
21	45.99	45.99	45.98	0.00098	0.01007	38.76	38.77	38.76	0.00212	0.01671
22	45.94	45.94	45.94	0.00144	0.00707	38.72	38.72	38.72	0.00171	0.00194
23	45.92	45.92	45.92	-0.00045	0.00626	38.71	38.71	38.71	0.00210	0.00339
24	45.91	45.91	45.91	-0.00059	0.00224	38.70	38.70	38.70	0.00074	0.00044

Note: DAP - DA price; DP - Dynamic price; SP - Settled price

3.2.2. Load distribution at the nodes resulting from DR

Influence of dynamic pricing and corresponding DR actions on the daily demand patterns and load factors at the two load nodes are depicted in Fig. 8. The plots on the left exhibit the initial ad-hoc demand patterns that are simulated to mimic the two peak demand patterns observed in real networks. Bars represent the fixed load. The red dotted line represents the ad-hoc total demand pattern combining the fixed and deferrable loads. The stable demand pattern achieved using DR is shown by the blue dotted line. Clearly, the deferrable portion of the initial demand pattern is redistributed from high to low demand periods by the DR actions, yielding a lower PAR (peak to average ratio) value. The plots on the right demonstrate the evolution of the load factor (ratio of average load to maximum load of a day) over the learning period. It is considered in the sample network problem that between 10% to 15% of its daily loads

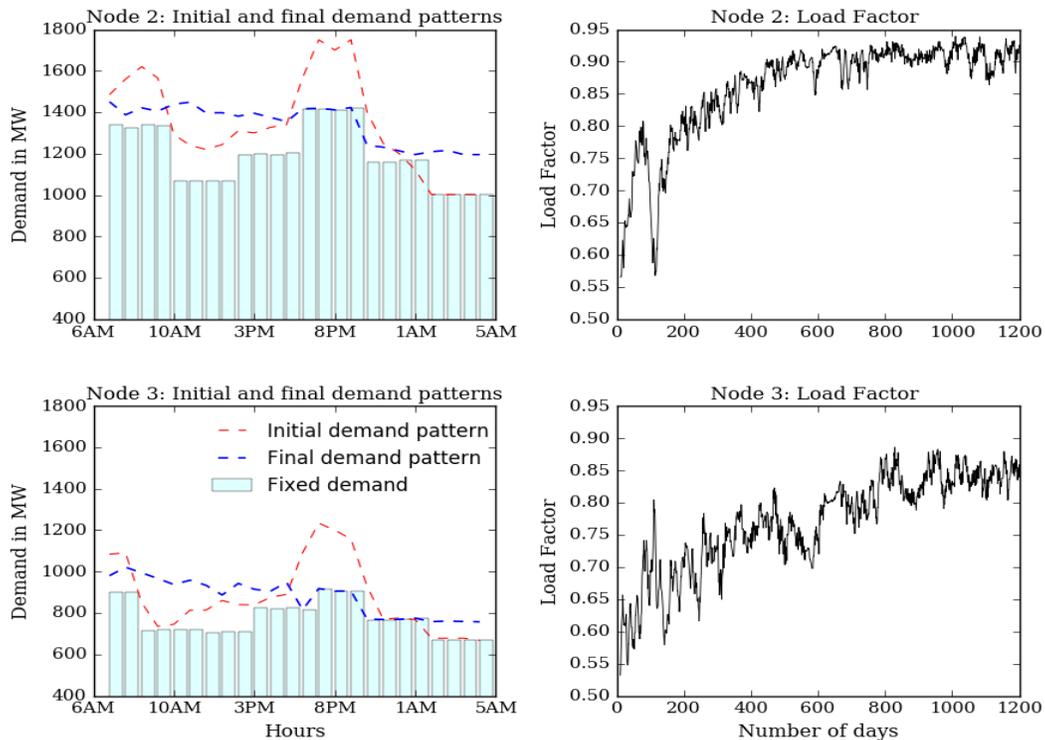


Fig. 8. Load pattern and corresponding load factors

are deferrable. Despite the small proportion of deferrable loads, due to the fluctuations in the DR actions in the initial learning stages, the load factors are at very low values near 0.55. As the learning progresses and the DR actions stabilize, the load factors improve to near 0.9.

3.2.3. Sensitivity of the proportion of deferrable loads

We studied the impact of the percentage of deferrable loads in the network on the average daily dynamic price and the load factor. Table 2 shows that even for a small level of deferrable load (5 - 7 %), a stable dynamic pricing and DR strategy improves the load factor by about 10% compared to the scenario with no DR. Also, the load factor and price improves with the increase in the percentage of deferrable loads. This shows that both power networks as well as the consumers could benefit by being more flexible, i.e., designating more of their loads as deferrable.

Table 2 Effect of the level of deferrable load on load factor and average dynamic price

% of Deferrable load participation	Node 2 (Aggregator 1)		Node 3 (Aggregator 2)	
	Load factor	Avg.Dynamic price \$/MWh	Load factor	Avg.Dynamic price \$/MWh
0 (no DR)	0.7678	-	0.7109	-
5-7	0.8588	48.47	0.806	40.92
10-12	0.9192	48.24	0.8607	40.75
14-16	0.946	48.16	0.87331	40.75

4. Conclusions

This paper presents a new data driven offline methodology that simulates power networks that operate with dynamic pricing and demand response. An application on a congested network shows that it is possible to obtain stable strategies for dynamic pricing and DR that can improve load balance in the network and reduce cost to the consumers. This helps to dispel the common apprehension that dynamic pricing and DR could increase electric power market volatility and reduce network reliability. It is also evident from our results that it is viable for SO to offer binding dynamic prices ahead of consumption. The methodology learns offline without disrupting actual network operations. For any change in the network configuration and other parameters (e.g., DA and RT price bids), the simulation can be rerun and new strategies can be learned. The sensitivity analysis shows that there is a financial motivation for the consumers to designate more of their loads as deferrable, as DR yields lower average prices of electricity.

Examination of the Bayesian demand prediction model (Fig. 5) shows that it is able to incorporate varying demand from DR actions in the learning process and improve its prediction to reduce forecasting error. In addition to effective demand forecasting, the other aspect of our methodology that helps the dynamic prices to reach stable values is the evolution of the π_t^{max} term in the robust DR model (see Fig. 6). A game theoretic optimization approach to obtain stable dynamic pricing and DR actions for a network

can be found in [34]. The findings of our data-driven methodology is similar to that in [34], which also obtains stable policies while maintaining the revenue neutral status of SO, balancing network load, and reducing cost to consumers.

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